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Topological term in the string representation of the Wilson loop in the dilute instanton gas approximation

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A topological term related to the number of self-intersections of the string world sheet is shown to emerge in the string representation of the Wilson loop in the dilute instanton gas. The coupling constant of this term turns out to be proportional to the topological charge of the instanton gas under consideration. [S0556-2821(98)08216-2]

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Recently, a string representation of the Wilson loop in the framework of the method of vacuum correlators [1–3] has been proposed [4,5]. Within this approach, the expansion of the Wilson loop in powers of the derivatives with respect to the string world-sheet coordinates has been performed. In the lowest orders, this yielded the Nambu-Goto and rigidity terms in the effective action, whose coupling constants were expressed in terms of the bilocal correlator of the gauge field strength tensors. In this way, the bare coupling constant of the rigidity term has been obtained to be negative, which is important (although not sufficient) for the stability of the string [6]. However, as has been explained in Ref. [7], if one considers this coupling constant as a running one, this could lead to the problem of crumpling of the string world sheet in the infrared region.

It is worth noting that quite recently, a possible solution to the problem of crumpling for the effective string theory emerging from D-dimensional compact QED (the so-called confining string theory [8]) has been put forward in Ref. [9]. There, it has been demonstrated that in the low-energy limit of this theory, for the case $D\to\infty$, the correlation function of two transversal fluctuations of the string world sheet has an oscillatory behavior at large distances. Such a behavior indicates that the world sheet is smooth rather than crumpled. One might expect that the same mechanism works in all gauge theories, whose confining phases admit a representation in the form of some effective string theory with a nonlocal interaction between the world-sheet elements. However, it is not obvious whether this mechanism can be extended to the non-Abelian case of gluodynamics, where in the nonlocal string effective action instead of the propagator of the Kalb-Ramond field, appearing in the Abelian case, stands the bilocal correlator of gluonic field strength tensors [4]. Though, according to the lattice data [10], the large distance asymptotic behavior of the latter is actually similar to the one of the Kalb-Ramond propagator, there nevertheless remain significant differences.

A possible solution of the problem of crumpling in gluodynamics has been proposed in Ref. [7]. It is based on the introduction of the so-called topological term, which is equal to the algebraic number of self-intersections of the string world sheet, into the string effective action. Then, by adjusting the coupling constant of this term, one can eventually arrange the cancellation of contributions into the string partition function coming from highly crumpled surfaces, whose intersection numbers differ by one from each other.

Thus it looks natural to address the problem of derivation of the topological term from the gluodynamics Lagrangian. Such a term has been recently derived in Ref. [11] for four-dimensional (4D) compact QED with an additional θ term. In the dual formulation of the Wilson loop in this theory (which is nothing else but the 4D confining string theory), the latter one occurred to be crucial for the formation of the topological string term. However, such a mechanism of generation of a topological term is difficult to work out in gluodynamics, due to our inability to construct the exact dual formulation of the Wilson loop in this theory. Therefore, it looks suggestive to search for some model of the gluodynamics vacuum, which might lead to the appearance of the topological term in the string representation of the Wilson loop in this theory. In the present paper, we shall elaborate on one such possibility. To this end, we shall make use of recent results concerning the calculation of the field strength correlators in the dilute instanton gas model [12]. There, it has been demonstrated that for the case of an instanton gas with broken CP invariance, the bilocal field strength correlator contains a term proportional to the tensor $\varepsilon_{\mu\nu\lambda\sigma}$. This term is absent in the case of a CP-symmetric vacuum, since it is proportional to the topological charge of the system $V(n_4-\overline{n}_4)$, where V is the four-volume of observation, and within the notations of Ref. [12], n_4 and $\overline{n}_4$ stand for the densities of instantons and antiinstantons (I's and $\bar{I}$'s for shortness), respectively. Similarly, we shall also work in the approximation of a dilute $I-\bar{I}$ gas with fixed equal sizes ρ of I's and $\bar{I}$'s.

Let us now briefly remind the reader of the main idea of Ref. [4]. According to this paper, the nonlocal string effective action associated with the surface of a minimal area $S_{\rm min}$, bounded by the contour of the Wilson loop, is defined as $\mathcal{A}_{\rm eff}=-\ln\langle W(S_{\rm min})\rangle$. Here the Wilson loop itself can be written within the method of vacuum correlators [1–3] as follows:

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$$\langle W(S_{\rm min})\rangle = \frac{1}{N_c^2-1}\mathrm{tr}\,\exp\left(-\int_{S_{\rm min}} d\sigma_{\mu\nu}(x)\int_{S_{\rm min}} d\sigma_{\lambda\sigma}(x')\times\langle F_{\mu\nu}(x,x_0)F_{\lambda\sigma}(x',x_0)\rangle\right), \qquad (1)$$

where $F_{\mu\nu}(x,x_0)$ is the so-called covariantly shifted strength tensor of the gauge field, which is related to the usual tensor $F_{\mu\nu}(x)\equiv F^a_{\mu\nu}(x)T^a_{\rm adj}$ as follows

$$F_{\mu\nu}(x,x_0)\equiv\Phi(x_0,x)F_{\mu\nu}(x)\Phi(x,x_0).$$

In this equation, $\Phi(x,x_0)$ stands for the parallel transporter factor of the gauge field defined on the straight-line contour, which goes from the fixed point x_0 to the point x. The bilocal correlator of the field strength tensors standing on the right-hand side (RHS) of Eq. (1) can be further parametrized by the two independent Lorentz structures, whose coefficient functions, denoted in Refs. [1–3] as $\mathcal{D}$ and $\mathcal{D}_1$, contain the main information about both nonperturbative and perturbative properties of the gluodynamics vacuum. These two functions decrease fastly at some distance T_g, which in Refs. [1–3] has been called the correlation length of the vacuum and according to the lattice data [10] is equal for the SU(3) case to 0.2 fm. Then, since the typical size r of the contour of the Wilson loop is of the order of 1.0 fm [13] in the confining regime under study, one can perform the expansion of the nonlocal action $\mathcal{A}_{\rm eff}$ in powers of $(T_g/r)^2$. This expansion is in fact nothing else but the expansion in powers of the derivatives with respect to the string world-sheet coordinates, mentioned in the beginning of the present paper.

The new structure arising in the bilocal correlator in the $I-\bar{I}$ gas reads [12]

$$\Delta\ \mathrm{tr}\langle F_{\mu\nu}(x,x_0)F_{\lambda\sigma}(x',x_0)\rangle = 8(n_4-\bar{n}_4)I_r\left(\frac{(x-x')^2}{\rho^2}\right)\varepsilon_{\mu\nu\lambda\sigma}. \qquad (2)$$

In Eq. (2), the asymptotic behavior of the function $I_r(z^2)$ at $z\ll 1$ and $z\gg 1$ has the following form:

$$I_r(z^2)\to\frac{\pi^2}{6} \qquad (3)$$

and

$$I_r(z^2)\to\frac{2\pi^2}{(z^2)^2}\ln z^2, \qquad (4)$$

respectively.

In what follows, we are going to present the leading term in the derivative expansion of the correction to the nonlocal string effective action

$$\Delta\mathcal{A}_{\rm eff}=-\ln\Delta\langle W(S_{\rm min})\rangle, \qquad (5)$$

where $\Delta\langle W(S_{\rm min})\rangle$ is a correction to the expression (1) for the Wilson loop, following from Eq. (2) in the CP-broken vacuum. Here we shall not be interested in calculating corrections to the Nambu-Goto and rigidity terms arising due to additional contributions from the $I-\bar{I}$ gas to the functions $\mathcal{D}$ and $\mathcal{D}_1$, which stand at Lorentz structures without the tensor $\varepsilon_{\mu\nu\lambda\sigma}$. According to Ref. [4], this can be easily done by carrying out the corresponding integrals of the functions $\mathcal{D}$ and $\mathcal{D}_1$ in this gas. Notice only that, as it has already been mentioned in Ref. [12], due to the reasons discussed in detail in Refs. [2] and [3], a correction to the string tension of the Nambu-Goto term obtained in such a way from the $I-\bar{I}$ gas contribution to the function $\mathcal{D}$ should be cancelled by the contributions coming from the higher cumulants in this gas.

Let us now turn to the expansion of the correction (5), emerging from the term (2), in powers of the derivatives with respect to the string world-sheet coordinates. First, one can see that since $\tilde{t}_{\mu\nu}t_{\mu\nu}=0$, where $t_{\mu\nu}$ is the extrinsic curvature tensor of the string world sheet, an analogue of the Nambu-Goto term for this correction vanishes. Then, similarly to Ref. [4], we get

$$\Delta\mathcal{A}_{\rm eff}=\alpha\nu+\mathcal{O}\left(\frac{\rho^6(n_4-\bar{n}_4)}{r^2}\right),$$

where

$$\nu\equiv\frac{1}{4\pi}\varepsilon_{\mu\nu\lambda\sigma}\int d^2\xi\sqrt{g}g^{ab}(\partial_a t_{\mu\nu})(\partial_b t_{\lambda\sigma})$$

is the algebraic number of self-intersections of the string world sheet and

$$\alpha=16\pi\rho^4(n_4-\bar{n}_4)\int d^2z z^2 I_r(z^2) \qquad (6)$$

is the corresponding coupling constant. Here g^{ab} stands for the induced metric tensor of the string world sheet, whose determinant is denoted by g.

Note that the averaged separation between the nearest neighbors in the $I-\bar{I}$ gas is given by $R=(n_4+\bar{n}_4)^{-1/4}$. According to phenomenological considerations one obtains for the SU(3) case, $\rho/R\simeq 1/3$ [14]; see also Ref. [15], where the ratio ρ/R has been obtained from direct lattice measurements to be $0.37-0.40$. R should then serve as a distance cutoff in the integral standing on the RHS of Eq. (6). Taking this into account, one gets from Eqs. (3), (4), and (6) the following approximate value of α:

$$\alpha\simeq(2\pi\rho)^4(n_4-\bar{n}_4)\left[\frac{1}{12}+\left(\ln\frac{R^2}{\rho^2}\right)^2\right], \qquad (7)$$

where the second term in the square brackets on the RHS of Eq. (7), emerging due to Eq. (4), is much larger than the first one, emerging due to Eq. (3).

In conclusion, we have found that in the $I-\bar{I}$ gas with a nonzero topological charge, there appears a topological term in the string representation of the Wilson loop. The coupling constant of this term is given by Eq. (7). Together with the Nambu-Goto and rigidity terms obtained in Ref. [4], this term forms the effective Lagrangian of the gluodynamics string following from the method of vacuum correlators.

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